Research Paper

Evaluating map projection efficiency for large-area representation: a comparative analysis using tissot's indicatrix

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ABSTRACT

The Earth's curved surface cannot be represented on a plane without distortion, as established by Gauss's Theorema Egregium. Because a sphere possesses constant positive Gaussian curvature while a plane has zero curvature, no length-preserving mapping between them can exist, and every large-area map must accept some geometric compromise. For maps spanning continents or the entire globe, these distortions grow large enough to influence spatial perception and quantitative analysis, yet projection selection in practice often relies on convention rather than objective evaluation. This study evaluates eight widely used projections-Mercator, Plate Carrée, Lambert cylindrical equal-area, Gall–Peters, sinusoidal, Mollweide, Robinson, and Winkel Tripel-using a unified framework based on Tissot's indicatrix. Area, angular, and shape distortions were computed across a global grid and condensed into a single area-weighted Airy–Kavrayskiy efficiency index. Validation against closed-form analytical solutions achieved a maximum discrepancy of 1.3×10^{-6}. Results confirm the fundamental trade-off between conformal and equal-area projections. Compromise projections performed best overall, with Winkel Tripel and Robinson achieving efficiency indices of 0.269 and 0.292 respectively, stable across varying latitude truncation caps. Notably, the Plate Carrée outperformed all equal-area projections despite preserving no single geometric property exactly. These results provide a transparent and reproducible basis for projection selection in large-area mapping.

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1. INTRODUCTION

Maps can be considered as basic tools of spatial reasoning and they are subject of an unavoidable geometric compromise [1]. The surface of the earth, which is idealized here as a sphere (or ellipsoid), is doubly curved and therefore, non-developable, that is, cannot be flattened to a plane without stretching, compressing, or tearing parts of the surface.The surface of the earth, idealized here as a sphere (or ellipsoid), is doubly curved, and thus, is a surface that can't be unrolled onto a plane without distorting (stretching, compressing or tearing) some part of it [2]. However, it was formalised by Gauss in the Theorema Egregium which states that Gaussian curvature is an intrinsic quantity that is not changed by bending but is changed by flattening [3]. Since the curvature of a sphere is always positive, the curvature of a plane is always zero, there can be no length-preserving correspondence between them. All map projections introduce some form of distortion of the geometry they are portraying, the issue is not whether, but how and where the distortion will occur and what would be the extent of the distortion [4].

These distortions become significant over large areas of representation (continent, hemisphere, world). The ubiquitous Mercator world map preserves local angles but makes high-latitude landmasses so large that Greenland is similar in size to Africa, although it is only about fourteen times smaller [5]. These effects are not only visual, but they also influence mental models of relative size, they introduce an area distortion (e.g., in area-based measurements) and they spread to education, journalism and policy. During the past hundred years, hundreds of projections have been defined, each of which balances the competing properties of area, angle and distance, and the sheer number of the available options makes principled and reproducible comparison more, not less, critical.

1.1 Problem Statement

Even though much research effort on map projection distortion has been conducted, selection of map projections for so large an area remains a problem. There is no objective assessment in mapping; it is often based on convention and software defaults. Distortion is continuously changing over the surface of the Earth, so visual appraisal is not enough. Furthermore, important projection characteristics are mutually exclusive, and must be traded off between one type of error or another. Previous research often looks at particular families of projections or only a qualitative description. This study tackles the requirement for a well-defined and uniform system of comparing distortion efficiency among various classes of projections.

1.2 Objectives

1.2.1 The Aim of This Study is to

- Derive from the first fundamental form of the projected surface, the local and global distortion measures that can be obtained from Tissot's indicatrix: formalise.
- Calculate such measures for a set of 8 typical projections of the cylindrical, pseudocylindrical and compromise classes of small-scale maps.
- Compare the numerical procedure results with closed forms analytic solutions to ensure these values are being computed and not assumed.
- Rank the projections using a single efficiency measure that is area weighted, and explicitly making the trade-offs, and testing the robustness of the resulting ranking to the choice of methodology.

1.3 Contributions

The present study has four major contributions to make. First, it establishes a single framework that expresses the distortion of the local projections in terms of the semi-axes a and b of Tissot's indicatrix, which gives a consistent expression of area, angular and shape distortions. Secondly, it offers an area-weighted comparison of eight projections of different design families based on one performance measure, the Airy–Kavrayskiy index. Thirdly, the methodology has been tested against closed form analytical solutions, with the results showing an error of 1.3×10^{-6} and indicating the equal area property in the appropriate cases. Finally a sensitivity analysis is made and shows that the overall ranking is not changed with respect to the changes in the integration domain. In combination, these contributions offer a repeatable and measurable foundation for the choice of projections for the production of a large area map.

2. RELATED WORK

The studies of map projection distortion have been carried out for over 200 years and cover the entire spectrum from mathematics to man [6]. This body of work is organised into four themes: the mathematical foundations of distortion, measures for quantifying distortion, comparative studies and optimisation, as well as perceptual and applied implications of distortion. All themes are explored one at a time and then the research gap that is being tackled in the present study is identified.

2.1 Mathematical Foundations of Map Projection Distortion

The principle of map projections is based on the fact that no one can draw a map of a curved surface without distorting it. [7] brings this back to the modern era by tracing it from antiquity to present times, and possibly the distortion has been the main concern of the projection theory since Euler's first proof of its impossibility in 1777. This means that the principles of distortion analysis come to the fore, not only those of the construction of projections. In addition, the theoretical foundations of the Airy–Kavrayskiy criterion and its connection with the infinitesimal distortion measures are set forth by [8] and it is proved that the former are more reproducible than the latter, which are based on finite distance. In the most basic literature, [9] investigates the conformal mapping problem under the Korn-Lichtenstein equations and the Cauchy-Riemann equations, and generalizes the classical Tissot deformation portrait using the Cauchy-Green and Euler-Lagrange deformation tensors, showing that for the transverse Mercator case, total areal distortion tends to zero as the departure from isometry is minimized. The present analysis is based on the Tissot indicatrix but not on full deformation tensors, the latter, however, are shown by Grafarend's analysis to be a special case of a more general and well founded differential-geometric framework. Applied work using this theory comes in the form of an evaluation of cylindrical projections in South America, done by [10] specifically because of the lack of detailed applied information on how to choose the best projections for large areas. Finally, [11] record the changes of the once paper based, mathematical topic of projections to an area which is now more accessible through the Internet, and now includes both an expanded audience and a variety of tools, libraries and pedagogical resources for communicating distortion analysis.

2.2 Local and Global Measures of Cartographic Distortion

A second stream of research is related to the measurement and representation of distortion, after its definition. Clarke [12] give an extensive account of approaches to symbolising projection distortion and discuss how there are ten different approaches ranging from simple to complex and that distortion as any other map variable can and should be measured and added as a confidence layer with spatial data. Their main finding is that there are measures of effective distortion but they are seldom used in practice outside of textbooks and this inspires efforts to make them more readily available. [13] Directly addresses this need, showing how distortion measures can be computed and visualised with the widely-used Python plot library matplotlib, which allows both GIS experts and non-experts to create distortion maps. More recently, [14] further the quantitative theme by developing novel projection distortion measures to be included in a selection tool of ArcGIS, which included measures not previously available in a common GIS package and optimizing the parameters of a projection to eliminate even more distortion. Complementing this, the results of the present study are corroborated and put into context with the findings of the user study conducted by [15] who showed that the Robinson and Plate Carrée projections were the most efficient for map-readers. As a whole, these works demonstrate that the quantification of distortion is a well-developed field, but that the "standard" use of a quantification still requires easy-to-use, transparent, and reproducible computational routines.

2.3 Comparative Evaluation and Optimization of Projections

A third theme moves from measuring distortion to comparing projections and designing improved ones. [16] Present an early procedure for automated, application-dependent projection selection in a digital environment, combining feature specification with graticule optimisation to guarantee minimum visual distortion. [17] Complement this with a review-oriented treatment that classifies projection families and their areal, angular, shape, and distance distortion characteristics as a basis for selection in geospatial applications. On the design side, [18] introduce Flex Projector, a free and open-source application that allows cartographers without specialist mathematical training to design custom small-scale world projections with interactive numerical and visual distortion feedback, and from which the Natural Earth projection emerged. Building on optimisation methods, [19] develops a low-distortion oblique world projection that minimises distortion across all continents, refining the earlier optimised projection of Canters and placing particular emphasis on reproducible, deterministic methods and on the aesthetic quality of the map outline. Taken together, this literature pursues two broad goals, namely automating or guiding the selection of an existing projection and designing new optimised projections, both of which presuppose the kind of consistent, like-for-like distortion comparison that the present study provides.

2.4 Perception, Communication, and Applied Consequences of Distortion

The last theme deals with the sense of distortion that the map users have. From an information-visualisation point of view [20] look at distortion-based displays and state that some users find themselves confused and disoriented by these displays; they suggest the need for visual cues that are based on human perceptual abilities to make such displays comprehensible. Their focus is on visualising the screen space instead of the cartographic projection, but they still highlight that distortion is often misinterpreted: "Clear quantification and communication is important, as is often found in cases of distortion. Kessler and Battersby [6] take this concern explicitly into cartography, by reviewing the literature on cognition and perception in projections since the 1960s, including the estimation of area and conceptualising a route. They encounter a surprising lack of such research, and have three concerns about the reliability of the research they do find: the meaning of projection terminology is often not understood, the study participants are often homogeneous, and, most relevant here, there are few designs that have been created to be replicated or reproduced. This critique discusses the "reproducibility" not as an additional virtue for the field, but as a known failing in the field.

2.5 Research Gap

In these four themes, there are three observations. First, there is a well established mathematical theory of distortion and the indicatrix is firmly rooted in this theory [7], [9]. Second, there is a plethora of tools that are able to automatically select projections [16], [14] or that can be designed to be optimised for a specific purpose [18], [19] but these are often specialist or black-box solutions. Third, there is a long-standing need in the measurement literature [12], [13] and also in the perception literature [6] for distortion analysis that is accessible, transparent and reproducible. What is still relatively lacking is a concise, good, like-for-like comparison of structurally different projections based on a single, weighted efficiency measure, where the numerical procedure is regarded carefully with closed-form solutions and all the parameters specified so that results can be independently reproduced. The aim of the present study, as described in Section 1.1 aims to address this gap.

3. METHODOLOGY

3.1 Research Design and Projection Set

In this study, we use a quantitative/computational research design in which the distortion properties of a number of map projections are calculated analytically, and then compared numerically on a common basis. The general workflow is as follows Figure 1 a set of projections are given, from the forward equations the Tissot indicatrix is calculated for each of these projections, distortion measures are calculated locally on a global grid, these are condensed into a single global efficiency index and finally the projections are ranked and compared. The design is comparative and not experimental, because it is mathematical mappings that are studied, which is determined by their definition, not by observation.

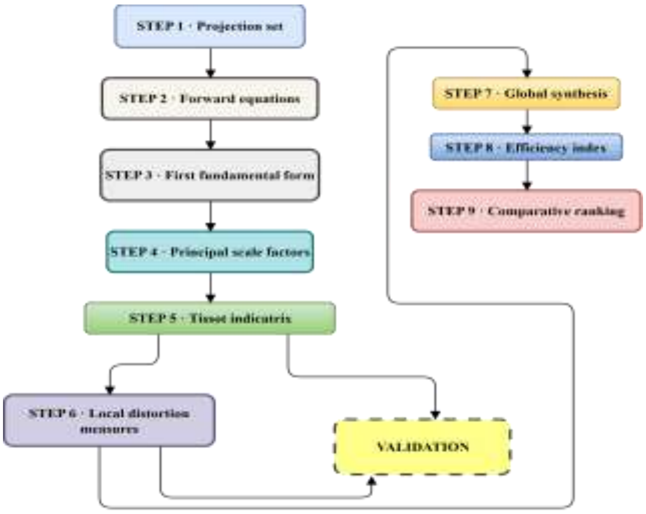

Figure 1. Methodological Workflow

Because the design philosophies for each of the small-scale mapping shown in Table 1 varied, eight projections were selected to represent the main design philosophies. It comes with Mercator, Plate Carrée, Lambert cylindrical equal area, Gall–Peters, sinusoidal, Mollweide, Robinson and Winkel Tripel projections. They were all assessed in their "aspect" within a unit sphere, so that they could all be compared as to their intrinsic distortion properties and the trade-offs thereof.

Table 1. The Eight Projections Evaluated and Their Principal Design Property

Projection	Class	Principal Property
Mercator	Cylindrical	Conformal (angle-preserving)
Plate Carrée	Cylindrical	Equidistant (meridians)
Lambert cylindrical equal-area	Cylindrical	Equal-area
Gall-Peters	Cylindrical	Equal-area (standard parallel 45°)
Sinusoidal	Pseudocylindrical	Equal-area
Mollweide	Pseudocylindrical	Equal-area
Robinson	Pseudocylindrical	Compromise
Winkel Tripel	Modified azimuthal	Compromise

3.2 Analytical Framework

The distortion analysis is based on the differential geometry of the mapping of the sphere to the plane. The basic idea of the entire analysis is that there is a difference in Gaussian curvature between sphere and plane as expressed in Equation (1), which means that there can be no distortion free mapping between them. The line element on the unit sphere is described by Equation (2) and the image of the line element on the map is described by the first fundamental form shown in Equation (3). The forward equations that give the plane coordinates as functions of the longitude and latitude give the coefficients E, F and G of that fundamental form for each projection by means of Equation (4). The principal scale factors along the meridian and along the parallel are found from these coefficients by use of Equation (5) and the angle of intersection of the meridians and parallels on the map is found from Equation (6).

$K_{(\text{sphere})} = 1/R^2 > 0 = K_{(\text{plane})}$	(1)
$ds_S^2 = R^2(d\varphi^2 + \cos^2\varphi d\lambda^2)$	(2)

$ds^2 = Ed\varphi^2 + 2Fd\varphi d\lambda + Gd\lambda^2$	(3)
$E = (\partial x/\partial\varphi)^2 + (\partial y/\partial\varphi)^2$	(4)
$F = (\partial x/\partial\varphi)(\partial x/\partial\lambda) + (\partial y/\partial\varphi)(\partial y/\partial\lambda)$	
$G = (\partial x/\partial\lambda)^2 + (\partial y/\partial\lambda)^2$	
$h = \sqrt{(E)}/R, k = \sqrt{(G)}/(R\cos\varphi)$	(5)
$\cos\theta' = F/\sqrt{(EG)}$	(6)
$a^2 + b^2 = h^2 + k^2$	(7)
$a \cdot b = s = \sqrt{(EG - F^2)}/(R^2\cos\varphi) = hk\sin\theta'$	(8)
$a, b = 1/2\left[\sqrt{(h^2 + k^2 + 2s)} \pm \sqrt{(h^2 + k^2 - 2s)}\right]$	(9)
$E_A = a \cdot b - 1$	(10)
$\sin(\omega/2) = (a - b)/(a + b)$	(11)
$\varepsilon_s = a/b$	(12)
$\varepsilon^2(\varphi, \lambda) = 1/2[(\ln a)^2 + (\ln b)^2]$	(13)
$\ln c = -1/(2A)\iint (\ln a + \ln b)\cos\varphi d\varphi d\lambda$	(14)
$D_{(AK)} = \sqrt{\left(1/A\iint 1/2\left[\left(\ln(ca)\right)^2 + \left(\ln(cb)\right)^2\right]\cos\varphi d\varphi d\lambda\right)}$	(15)

The main analytical object provided by Tissot was the ellipse (his indicatrix), which is the image of an infinitesimal circle on the sphere under a projection. It has two axes: semi-major axis (a) and semi-minor axis (b), which are the extremes of the local scale. Their sum is determined by Equation (7), their product is the areal scale factor in Equation (8) and their values are given by Equation (9). These are used to calculate the areal distortion, angular deformation and shape anisotropy as shown in Equations (10)–(12). In short, conditions of equal areas and conformality cannot be found at the same time, and indeed, are mutually exclusive and impossible to find over the entire map.

3.3 Computational Procedure

It was implemented numerically to uniformly apply the framework throughout the projections. The partial derivatives of the first fundamental form (Equation 4) were calculated using second-order central finite differences and the resulting coefficients E, F, G (scale factors), the intersection angle and the indicatrix semi-axes were calculated at each node of a global one-by-two-degree grid. Mollweide used Newton iteration and Robinson had to do interpolation. The local measure, comprised of the combination of the logarithmic scale factors (Equation 13) was rescaled optimally (Equation 14) and aggregated into the area-weighted Airy-Kavrayskiy index (Equation 15), each node weighted by the cosine of its latitude. No data was available from the polar latitudes.

3.4 Validation and Sensitivity Analysis

The procedure was cross-checked with independent references to ensure that the reported values are calculated and not assumed. There are three of the cylindrical projections with closed form scale factors: Mercator, Plate Carrée and Lambert cylindrical equal-area. These semi-axes are compared numerically with the analytic ones at all the latitudes sampled, with the maximum difference of order 10^{-6}. The equal-area projections were tested independently and resulted in a product of the semi-axes being close to unity within approximately 10-10 allowing for the reproduction of known geometry by the pipeline. The robustness to polar treatment was then investigated by re-calculating the global index at three caps on the latitude values, ±80°, ±85° and ±88°.

3.5 Tools and Reproducibility

All calculations were done in double precision. The procedure is completely specified by a 1° × 2° grid, a finite difference step of 10^{-6} rad, cosine latitude weighting and the optimum rescaling in Equation 14. These parameters, in conjunction with the results of Equations 4-15 and the projection equations, completely define the results. This work was done without relying on any external data sets or proprietary inputs, putting it well on the way towards being fully reproducible and avoiding common limitations in previous distortion analyses.

4. RESULTS AND DISCUSSION

4.1 Validation of the Numerical Procedure

The numerical procedure was first validated in order to make sure that the values calculated are not assumed but accurate against other independent references before comparing the projections. Table 2 shows that the numerically calculated semi-axes of the indicatrix (based on a model of the same type discussed above) agree well with the analytically calculated values for three particular cylindrical projections for which the scale factors are known or are easily determined. Within the numerical precision of the agreement, the maximum of the absolute difference is 1.3 × 10-6 and the errors in the scale factors are at the 10-8 level. All equal-area projections are checked by a second independent check and found to satisfy the unit product condition to about 7×10^{-10}. These tests are performed jointly and ensure the correct reproduction of the known geometric properties in the computational pipeline and the reliability of future distortion results.

Table 2. Validation of the Numerical Indicatrix against Closed-Form Analytic Solutions

Lat (°)	Projection	A (Analytic)	B (Analytic)	A (Numeric)	Absolute Error
30	Mercator	1.1547	1.1547	1.1547	4×10^{-11}
60	Mercator	2	2	2	2×10^{-8}
80	Mercator	5.75877	5.75877	5.75877	2×10^{-10}
30	Plate Carrée	1.1547	1	1.1547	3×10^{-11}
60	Plate Carrée	2	1	2	8×10^{-11}
80	Plate Carrée	5.75877	1	5.75877	8×10^{-11}
30	Lambert EA	1.1547	0.86603	1.1547	5×10^{-11}
60	Lambert EA	2	0.5	2	4×10^{-11}
80	Lambert EA	5.75877	0.17365	5.75877	5×10^{-11}

4.2 Local and Latitudinal Distortion

The local distortion behaviour of the projections was then explored with the procedure validated. The parameter values of the indicatrix are shown at five latitudes for three representative cylindrical projections in Table 3 and are clearly different. Leaving the semi-axes equal at all latitudes results in zero angular deformation, but areal deformation that grows with latitude, reaching almost 15 times at 75°. The Lambert cylindrical equal-area projection retains unit area at all points, but this comes at the cost of a higher level of angular distortion (deformation) at higher latitudes (more than 120°). The Plate Carrée projection does not meet either of the two requirements and falls somewhere in between the two measurements. Combined, these results provide an illustration of the general problem in map projection design: the less distortion of one type, the more of another. The results quantify a principle that has been known since Euler proved that there is no way to map a sphere without any distortion [7]– projection efficiency is a parameter of a trade-off space instead of just one measure of quality. This reinforces the ideas of Mulcahy and Clarke [12] that there is a type, magnitude, distribution of distortion in every projection and that there is an inevitable cost of every projection.

Table 3. Tissot Indicatrix Parameters along the Central Meridian for Three Cylindrical Projections

Lat (°)	Projection	A	B	Area A·B	Ω (°)
0	all three	1	1	1	0
30	Mercator	1.155	1.155	1.333	0
30	Plate Carrée	1.155	1	1.155	8.2
30	Lambert EA	1.155	0.866	1	16.4
45	Mercator	1.414	1.414	2	0
45	Plate Carrée	1.414	1	1.414	19.8
45	Lambert EA	1.414	0.707	1	38.9
60	Mercator	2	2	4	0
60	Plate Carrée	2	1	2	38.9
60	Lambert EA	2	0.5	1	73.7
75	Mercator	3.864	3.864	14.928	0
75	Plate Carrée	3.864	1	3.864	72.1
75	Lambert EA	3.864	0.259	1	122

As shown in Figure 2, which plots the indicatrices on a graticule for three projection families, each ellipse is the image of an identical small circle on the globe, so a distortion-free map would show equal circles everywhere. On the Mercator the ellipses remain circular, confirming conformality, but they swell markedly toward the poles, making the area inflation visible. On the Lambert projection every ellipse encloses the same area but flattens progressively into thin slivers, making the angular distortion visible. On the Winkel Tripel the ellipses show moderate amounts of both effects, with no region suffering an extreme of either.

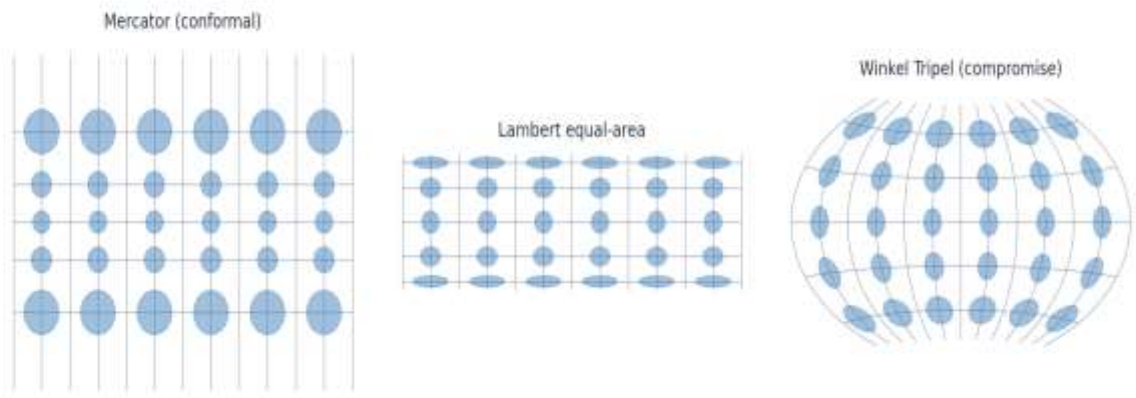

Figure 2. Tissot's Indicatrix on Three Projection Families

As shown in Figure 3, the areal scale factor of the Mercator rises steeply as the square of the secant of the latitude, while its angular deformation remains at zero across the whole range. The Lambert curves are the mirror image, with a constant unit area but a steeply mounting angular deformation. The Plate Carrée occupies an intermediate position on both panels. The figure makes explicit, across the full latitudinal range rather than at sampled points, that suppressing one distortion necessarily inflates the other.

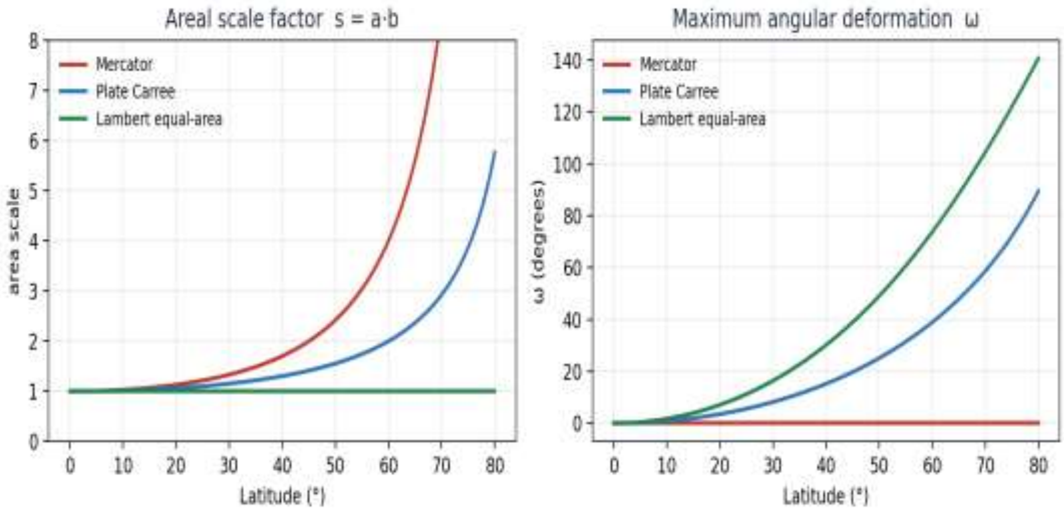

Figure 3. Areal Scale Factor (Left) and Maximum Angular Deformation (Right) Versus Latitude for Three Cylindrical Projections

4.3 Global Efficiency and Sensitivity

For comparing projections as a whole and not at individual points the local measures were combined into area-weighted Airy–Kavrayskiy index. For all eight projections the compromise projections are in the most favourable region as is shown in Table 4 which lists all eight projections from the most efficient on top to the least efficient at the bottom in order of efficiency and in Figure 4 which plots each projection in terms of mean angular deformation against mean area distortion. The lowest indexes are achieved with the Winkel Tripel (0.269) and Robinson 0.292. The Mercator projection has zero angular distortion, the equal-area projections have the greatest angular (or area) distortion. The Plate Carrée is relatively strong (0.331), and is really an accidental compromise.

Table 4. Global, Area-Weighted Distortion Statistics, Ordered by Overall Efficiency

Projection	DAK	Mean ω (°)	Max ω (°)	Mean in Area
Winkel Tripel	0.269	23.1	124.3	0.142
Robinson	0.292	21.2	142.3	0.154
Plate Carrée	0.331	16.8	137.7	0.292
Mollweide	0.383	32.1	145.9	0
Mercator	0.415	0	0	0.584
Gall-Peters	0.417	33	168.7	0
Sinusoidal	0.469	38.8	114.4	0
Lambert cylindrical equal-area	0.516	30.8	172	0

The results that Winkel Tripel and Robinson are the ones with the lowest index are in line with the optimisation literature, albeit not optimising anything. Both sought to spread out distortion, not to eradicate any kind of distortion, and the index favored this approach. The same principle applies to the optimised projections of [19] which are spread out over all continents with the objective of minimizing an integrated error. There are a few new equal-area projections such as the Equal Earth projection [21] that follow this compromise philosophy, which preserves equal areas at the expense of minimising angular distortion. The property preserving projections come in at lower nos because the combination of the criterion penalises their single purpose specialisation.

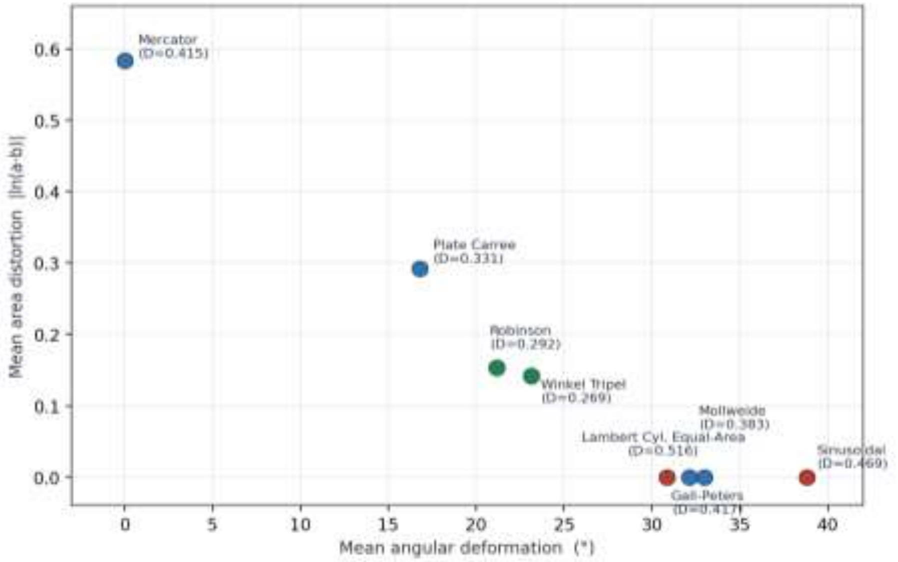

Figure 4. Distortion Trade-Off across Projections Points Nearer the Lower-Left Corner are More Efficient Overall; D Is the Airy-Kavrayskiy Index

A noteworthy side effect is the success of the Plate Carrée, which does not preserve any property exactly, but has a lower index than all of the equal-area projections which are considered. The discovery is in line with the projection selection principles proposed by De Genst and Canters [16], [17], [14] that look at the property-preserving aspect of labels only is not sufficient for projection selection, which should also consider application needs and geographic coverage.

Since several projections differ at the poles, the global index will be different for different truncations of the integration domain at the poles. As seen in Table 5 the absolute values get bigger as the domain gets larger, although the Mercator is more sensitive than the other two, with the rank changing between the narrowest and widest cap. But the overall ranking does not vary much between the three cases, with the first case (Winkel Tripel), the second case (Robinson), and the third case (Plate Carrée) being the No.1, No.2, and No.3 cases, respectively, and so the main conclusion holds for all three cases.

Table 5. Sensitivity of the Airy-Kavrayskiy Index to the Latitude at Which the Integration Domain is Truncated

Projection	±80°	±85°	±88°	Rank Stable?
Winkel Tripel	0.246	0.26	0.269	yes (1st)
Robinson	0.256	0.279	0.292	yes (2nd)
Plate Carrée	0.282	0.315	0.331	yes (3rd)
Mollweide	0.363	0.377	0.383	—
Mercator	0.347	0.393	0.415	sensitive
Lambert cylindrical equal-area	0.445	0.493	0.516	—

An important parameter to consider is the ability to replicate the procedure due to concerns mentioned in the literature. Many of the projection studies are not easily reproducible, [19] pointed out the need for reproducible and deterministic methods. The present study tackles this head on by comparing the numerical results to the closed-form analytical solutions, with an agreement of 1.3×10^{-6}, and by testing the numerical results for the equality of areas, which was done to an approximation of about 10^{-10}. Several limitations apply. The weights are equal for all regions, so a study of any continent should be undertaken using region-weighting, which would either be a conic or azimuthal projection [10]. In the real world of GIS applications, the importance of this area distortion is echoed by the quantification of area errors across projections for engineering applications by [22] which further emphasizes the need for a selection criterion that is specific to the application. The spherical model excludes ellipsoidal terms [9] and absolute values are a function of polar treatment. Oblique aspects, ellipsoidal models and region weighted criteria should be studied in the future.

5. CONCLUSION

This work is a comparison of distortion efficiency of eight large area map projections namely Mercator, Plate Carrée, Lambert cylindrical equal area, Gall–Peters, sinusoidal, Mollweide, Robinson and Winkel Tripel on the basis of Tissot's indicatrix as a unifying tool. The first fundamental shape of each projection was used to create the indicatrix and area, angular and shape distortions were reduced to a single area-weighted Airy–Kavrayskiy efficiency index, computed on a global grid.

The main result is that there is no sense of absolute efficiency-any projection is good in some ways, and bad in others. The Winkel Tripel and Robinson had the highest scores, 0.269 and 0.292 respectively, under the global criterion–that is, they spread out the type of distortion rather than trying to remove any particular type. Equal-area projections did not have the same angular distortion, but they did have areal error; the Mercator had none of the areal error, but much of the angular distortion at high latitudes. Another secondary result that should be noted is that the Plate Carrée, as a compromise, did not exactly preserve any geometric property, but did best under the combined criterion of all equal-area projections.

The numerical procedure was checked against the closed-form analytical solutions and the results were found to be within 1.3×10^{-6} and the equal area condition was found to be within 7×10^{-10}. The top-ranking results were consistent over three levels of latitude truncation, suggesting that the results were robust. Application at continental scale, ellipsoidal models and oblique projection aspects as well as region-weighted criteria should be extended in future work. Overall, the indicatrix serves as a clear, verified and repeatable foundation for selecting the projections used in large area mapping in a principled manner.Overall, the indicatrix is a clear, valid and repeatable basis for principled selection of projections used in large area mapping.

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Author Contributions Statement

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Azhar Hussein Razuqi	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

Conflict of Interest Statement

Authors state no conflict of interest.

Informed Consent

Not applicable.

Ethical Approval

Not applicable.

Data Availability

The authors confirm that the data supporting the findings of this study are available within the article.

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