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An Advanced Approach to HFP-Soft Set Based Decision- Making in Uncertain Environment

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Abstract: Uncertainty, vagueness and the representation of imperfect knowledge have been a problem in many fields of research, such as environmental science, artificial intelligence, network and communication, signal processing, machine learning, computer science, information technology, as well as medical science, economics, and engineering. Decision-making methods (simply, DMMs) based on hesitant fuzzy sets (HFSs) and soft sets (SSs) have recently received a lot of attention. Karamaz and Karaaslan (2021)proposed an approach to hesitant fuzzy parameterized soft set (HFPSS) based decision making using score function on hesitant fuzzy elements (HFEs) in HFPSS, with the goal of using HFSs and SSs more effectively for uncertainty issues prevalent in most real-world issues, but it has some limitations in solving HFPSS based decision-making problems (briefly, DMPs). In this study, we demonstrated with a real-life example that the Karamaz-Karaaslan DMM is insufficient to solve HFPSS based DMPs and we have proposed an advanced and adjustable DMM for solving HFPSS based DMPs in uncertain environment. Some real-life examples are provided to demonstrate the validity of our proposed modified DMM.

Keywords: Decision Making, Soft Set, Hesitant Fuzzy Set, HFPSS.

1. INTRODUCTION

Soft set theory (SST) was first presented by Molodtsov (1999)as a fundamental and useful mathematical method for dealing with complexity, unclear definitions, and unknown objects (elements). Since there are no limitations to the description of elements in SST, researchers may choose the type of parameters that they need, significantly simplifying DMPs and making it easier to make decisions in the absence of partial knowledge, it is more effective. While several mathematical models for dealing with uncertainties are available, such as

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operations analysis, probability theory, game theory, fuzzy set theory (FST), rough set theory, and interval-valued fuzzy (IVF) set theory, each of these theories has its own set of problems. Furthermore, all of these theories lack parameterization of the tools, which means they can't be used to solve problems, especially in the economic, environmental, and social realms. In the sense that it is clear of the aforementioned difficulties, SST stands out.The SST is extremely useful in a variety of situations. Molodtsov (1999) developed the basic results of SST and successfully applied it to a variety of fields, together with the smoothness of functions, operations analysis, game theory, Riemann integrations, the notion of probability, and so on. Later, Maji et al. (2003) presented several new SST concepts, such as subset, complements, union, and intersection, as well as their implementations in DMPs. Ali et al. (2009) identified some more operations on SST and demonstrated that De Morgan's laws apply to these new operations in SST. To solve the DMPs, Maji et al. (2002) used SST for the first time. Recently, several authors later looked into the more broad properties and applications of SST. Alcantud and Santos-García (2017) presented a new criterion for SST based DMPs under incomplete information. Deli and Karaaslan (2019) introduced the theory of Bipolar FPSS with applications in DMPs. Chen et al. (2020) suggested the notions generalized vague N-SSs in group DMPs and Dalkılıç (2021) proposed a novel approach to SST in DMPs under uncertainty.

The idea of the FST was started by Zadeh (1965), thereafter, many new approaches and ideas have been offered to deal with imprecision and ambiguity, such as the HFSs (Torra 2010), intuitionistic fuzzy sets (Atanassov, 1986), and so on (Akram et al. 2020, Mohammed and Abdulkareem 2020). FST has a wide range of applications, including databases, neural systems, pattern recognition, medicine, fuzzy modelling, economics, and multicriteria DMPs. Torra (2010) first introduced the theory of HFSs and later on, Torra and Narukawa (2009) studied more results on HFSs and used it in DMPs. Rodryguez et al. (2014) discussed the current state of HFSs and their potential directions. Distance and similarity measurements for HFSs were reported by Xu and Xia (2011). Xia and Xu (2011, 2017) proposed hesitant fuzzy (HF) information aggregation in DMPs and also, studied some properties of HFSs. Zhang (2013) presented HF-power aggregation functions as well as their utilizations in MCGDM. Zhu et al. (2014) derived a ranking from HF-preference relations in group DMPs. Bedregal et al. (2014) defined the notions of aggregation operators on typical HFEs with the action of automorphisms. Thereafter, Zhang et al. (2014) defined induced generalized HF-functions with their utilizations in MCGDM. Yu et al. (2016) suggested a dual HF-group DMM as well as its utilization in supplier selection. Ren and Wei (2017) proposed an MCDM algorithm with a prioritization relationship and dual HF-decision information. In a hesitant probabilistic fuzzy environment, Xu and Zhou (2017) demonstrated consensus building using a group of DMs. Liu and Zhang (2017, 2017a) suggested an extended MCDM technique using neutrosophic HF-information and also, proposed another MCDM technique using neutrosophic HF-heronian mean aggregation functions. Liang and others (2017, 2020) suggested three-way decisions using decision-theoretic rough sets with dual HF-information and proposed risk appetite dual HF-three-way decisions with TODIM. Alcantud and Torra (2018) presented some decomposition theorems with extension concepts for HFSs. Zhang et al. (2018) provided the notion of HF-linguistic rough set on two universes structure and its applications. Chen et al. (2018) presented distance measures for higher order dual HFSs.

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Fatimah and Alcantud (2018) expanded the idea of dual HFSs, and Kakati et al. (2019) presented the notion of interval neutrosophic HF-Einstein Choquet integral function for MCDM. Naz and Akram (2019) suggested anovel DMM based on HFSs and graph theory and Song et al. (2019) proposed an improved model learning method of the Bayesian network with the help of HF-information flow. Alcantud and Giarlotta (2019) studied the necessary and possible HFSs as well as proposed a novel model for group DMPs. Ozlu and Karaaslan (2019) developed the idea of distance measures for type 2 HFSs as well as their utilizationsin MCGDM problems. The HF-linguistic portfolio structure with changing risk appetite was presented and shown in DMPs by Zhou and Xu (2019). Xue et al. (2019) proposed using the evidential reasoning approach with multi-scale HF-linguistic information to assess the threat of landslide dams. Later, Zhang et al. (2020, 2020a) presented the ideas of multi-granularity three-way decisions on two universes with adjustable HF-linguistic multigranulation decision-theoretic rough sets and interval-valued HF-multi-granularity three-way decisions in consensus processes, as well as their applications in MCGDM.As an HF-linguistic MCDM approach, Ozkan et al. (2020) evaluated academic department websites using SEO criteria, while Hao and Chiclana (2020) proposed a DMM for hesitant fuzzy linguistic group DMPs.
Maji et al. (2001) described a mixture of FST and SST called fuzzy soft set (FSS).An FSS-based DMM was proposed by Roy and Maji (2007)to solve FSSs based real-life applications. The applications of FSSs have been gradually concentrated by using these concepts.Feng et al. (2010) introducing aflexible DMM for solving FSSs based real-life DMPs and Çağman et al. (2010)presented the theories of FPFSSs as well as their applications in DMPs. Wang et al. (2014)presented the definition of hesitant-FSS and proposed its applications in MCDM. Wei and Zhao (2013) introduced the induced hesitant IVF Einstein aggregation functions with their utilizations in MCDM and Wei et al. (2013) presented some more hesitant IVF aggregation functions as well as their utilizations in MCDM. Dey and others (2015, 2020) defined some new topological structures on hesitant multi- FSS and suggested a new approach to hesitant multi-FSS based DMPs. Zhang et al (2015) studied some more results on interval-valued hesitant-FSSs, and later on, Wei (2016) suggested the notions of interval valued HF-uncertain linguistic aggregation functions in MCDM. Qi et al. (2016) suggested an MCGDM using generalized power aggregation functions under interval-valued dual HF-linguistic atmosphere. Peng and Dai (2017) suggested some HF-soft DMMs using COPRAS, MABAC, and WASPAS with combined weights. Al-Qudah and Hassan (2018) presented the theory of complex multi-FSS as well as studied its entropy and similarity measure. Akram and Adeel (2019) proposed the TOPSIS method for MCGDM based on interval-valued HF-N-soft atmosphere. Later on, Akram et al. (2019, 2019a) presented group DMMs using hesitant N-SSs and defined HF-N-SSs as well as a novel technique with utilization in DMPs. Based on revised aggregation operators, Peng and Li (2019)suggested a method for solving Hesitant FSS based real-life DMPs. Li et al. (2019) presented the notions of generalized hesitant FSSs and their applications in DMPs. Petchimuthu et al. (2020)defined the mean functions as well as generalized products of fuzzy soft matrices and discussed their utilizations in MCGDM. Paik and Mondal, (2021) introduced a distance-similarity technique for solving FST and FSSs based DMPs. Paik and Mondal (2021a)had shown the representation and utilizations of FSSs in a type-2 atmosphere. Gao and Wu (2021) defined filter and its applications in topological spaces formed by FSSs. Dalkılıç and Demirtaş (2021) introduced the idea of bipolar fuzzy soft D-metric spaces. Dalkiliç (2021) defined topology

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on virtual fuzzy parameterized-FSSs. Bhardwaj and Sharma (2021) described an advanced uncertainty measure using FSSs and shown its application in DMPs.

Decision-making methods (simply, DMMs) based on hesitant fuzzy sets (HFSs) and soft sets (SSs) have recently received a lot of attention. Karamaz and Karaaslan (Journal of Ambient Intelligence and Humanized Computing, 2021) proposed an approach to hesitant fuzzy parameterized soft set (HFPSS) based decision making using score function on hesitant fuzzy elements (HFEs) in HFPSS, with the goal of using HFSs and SSs more effectively for uncertainty issues prevalent in most real-world issues, but it has some limitations in solving HFPSS based decision-making problems (briefly, DMPs). In this study, we demonstrated with a real-life example that the Karamaz-Karaaslan DMM is insufficient to solve HFPSS based DMPs. As a result, in this paper, we have defined the notions of root mean square operator (simply, RMSO) and geometric mean operator (simply, GMO) on HFEs in HFPSS, and using these two operators we have proposed an advanced and adjustable DMM for solving HFPSS based DMPs. We have also introduced two types of level fuzzy parameterized soft sets (level-FPSSs) as root mean square level-FPSS (RMS-level-FPSS) and geometric mean level-FPSS (GM-level-FPSS) and deduced their induced fuzzy set (IFS), using our newly defined two novel operators RMSO and GMO. The novelty of our proposed DMM is the concepts of RMSO and GMO rather than score function, which make our DMM more stable and more feasible than the Karamaz-Karaaslan DMM. The second uniqueness is that our DMM can be adjusted, whereas the Karamaz-Karaaslan approach cannot. Another difference is that we solve a real-life DMP using our suggested DMM, which impossible to solve using the Karamaz-Karaaslan method. Some real-life examples are provided to demonstrate the validity of our proposed modified DMM.

2. PRELIMINARY

Let us consider Ψ_{US} represents the starting universe, Φ_{PS} represents a nonempty parameter set and $pow(\Psi_{US})$ means the power set of Ψ_{US}.

Definition 2.1 (Zadeh 1965)A FS μ_{FS} on Ψ_{US} is a set with a structure $\mu_{FS} = \{(\psi, \mu_{FS}(\psi)) : \psi \in \Psi_{US}\}$, where the real-valued function $\mu_{FS} : \Psi_{US} \to [0, 1]$ is said to be the membership function and $\mu_{FS}(\psi)$ is called the degree of membership for each object $\psi \in \Psi_{US}$.

Definition 2.2(Xia and Xu 2010) A HFS on Ψ_{US} is a set with a structure $\mu_{HFS} = \{(\psi, \mu_{HFS}(\psi)) : \psi \in \Psi_{US}\}$, and it is defined by the terms $\mu_{HFS}(\psi)$ when applied to Ψ_{US}, where $\mu_{HFS}(\psi)$ is a collection of multiple values in the range [0, 1], reflecting the possible membership degrees for each member $\psi \in \Psi_{US}$ and $\mu_{HFS}(\psi)$ is called HFE.

Definition 2.3 (Xia and Xu 2010) Let $\mu_{HFS}=\{(\psi,\ \mu_{HFS}(\psi)):\ \psi\in\Psi_{US}\}$ be an HFS and $\mu_{HFS}(\psi)$ be a HFE of HFS μ_{HFS}. Then $r(\mu_{HFS}(\psi))=\frac{1}{|\mu_{HFS}(\psi)|}\sum_{\mu_k\in\mu_{HFS}(\psi)}\mu_k$, where r is said to be the score function and $r(\mu_{HFS}(\psi))$ is known as score value of $\mu_{HFS}(\psi)$.

Here $|\mu_{HFS}(\psi)|$ denotes the number of values in $\mu_{HFS}(\psi)$.

Definition 2.4 (Molodtsov 1999) A SS over the nonempty universe Ψ_{US} is a pair (λ_{SS},Φ_{PS}), where λ_{SS} is a mapping defined by $\lambda_{SS}:\Phi_{PS}\to pow(\Psi_{US})$. Thus, a SS (λ_{SS},Φ_{PS}) over Ψ_{US} can be represented as $(\lambda_{SS},\Phi_{PS})=\{(\varphi,\lambda_{SS}(\varphi)):\varphi\in\Phi_{PS}\}$.

Definition 2.5 (Çağman et al. 2010) A FPSS λ_{FPSS} on Ψ_{US} is a set with a structure $\lambda_{FPSS}=\{((\varphi,\mu_{FS}(\varphi)),\lambda_{SS}(\varphi)):\varphi\in\Phi_{PS}\}$, where $\mu_{FS}:\Phi_{PS}\to[0,\ 1]$ and $\lambda_{SS}:\Phi_{PS}\to pow(\Psi_{US})$ are functions.

Definition 2.6 (Karamaz and Karaaslan 2021) A HFPSS λ_{HFPSS} on Ψ_{US} is a set with a structure $\lambda_{HFPSS}=\{((\varphi,\mu_{HFS}(\varphi)),\lambda_{SS}(\varphi)):\varphi\in\Phi_{PS}\}$, where λ_{SS} is a mapping given by $\lambda_{SS}:\Phi_{PS}\to pow(\Psi_{US})$ and $\mu_{HFS}(\varphi)$ is a collection of multiple values in the range [0, 1], reflecting the possible membership degrees for each member $\varphi\in\Phi_{PS}$ and called HFE in λ_{HFPSS}.

Assume that, in this research paper $HFE(\Phi_{PS})$ means the collection of all HFEs on Φ_{PS}.

Example 2.7 Assume $\Psi_{US}=\{\psi_1,\psi_2,\psi_3,\psi_4,\psi_5,\psi_6\}$ be the set of the universe and $\Phi_{PS}=\{\varphi_1,\varphi_2,\varphi_3,\varphi_4,\varphi_5,\varphi_6\}$ be the set of parameters associated with the universes mentioned above. Let

$$\lambda_{SS}(\varphi_1)=\{\psi_1,\psi_2\},\ \lambda_{SS}(\varphi_2)=\{\psi_1,\psi_4,\psi_5,\psi_6\},\ \lambda_{SS}(\varphi_3)=\{\psi_2,\psi_3,\psi_5\},$$
$$\lambda_{SS}(\varphi_4)=\{\psi_1,\psi_2,\psi_5,\psi_6\},\ \lambda_{SS}(\varphi_5)=\{\psi_3,\psi_4,\psi_6\},\ \lambda_{SS}(\varphi_6)=\{\psi_3,\psi_4\},$$

and

$$\mu_{HFS}(\varphi_1)=\{0.2,0.6\},\ \mu_{HFS}(\varphi_2)=\{0.2,0.3,0.4,0.7\},\ \mu_{HFS}(\varphi_3)=\{0.2,0.4,0.6\},$$
$$\mu_{HFS}(\varphi_4)=\{0.2,0.3,0.5,0.6\},\ \mu_{HFS}(\varphi_5)=\{0.3,0.5\},\ \mu_{HFS}(\varphi_6)=\{0.3,0.4,0.5\}.$$

Then an HFPSS λ_{HFPSS} on Ψ_{US} can be written as

$$\lambda_{HFPSS}=\left\{\begin{array}{l}(\varphi_1,^{\{0.2,0.6\}}\{\psi_1,\psi_2\}),\ (\varphi_2,^{\{0.2,0.3,0.4,0.7\}}\{\psi_1,\psi_4,\psi_5,\psi_6\}),\ (\varphi_3,^{\{0.2,0.4,0.6\}}\{\psi_2,\psi_3,\psi_5\}),\\(\varphi_4,^{\{0.2,0.3,0.5,0.6\}}\{\psi_1,\psi_2,\psi_5,\psi_6\}),\ (\varphi_5,^{\{0.3,0.5\}}\{\psi_3,\psi_4,\psi_6\}),\ (\varphi_6,^{\{0.3,0.4,0.5\}}\{\psi_3,\psi_4\})\end{array}\right\}.$$

Definition 2.8 (Karamaz and Karaaslan 2021) Let $\lambda_{HFPSS}=\{((\varphi,\mu_{HFS}(\varphi)),\lambda_{SS}(\varphi)):\varphi\in\Phi_{PS}\}$ is an HFPSS on Ψ_{US}. The reduced HFPSS(RHFPSS) of an HFPSS λ_{HFPSS} is denoted as λ'_{RHFPSS} and defined by

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$\lambda'_{RHFPSS} = \{((\varphi, r(\mu_{HFS}(\varphi))), \lambda_{SS}(\varphi)) : \varphi \in \Phi_{PS}\}$, where $r(\mu_{HFS}(\varphi))$ is score value of HFE $\mu_{HFS}(\varphi)$ in λ_{HFPSS}, i.e., $r(\mu_{HFS}(\varphi)) = \frac{1}{|\mu_{HFS}(\varphi)|} \sum_{\mu_k \in \mu_{HFS}(\varphi)} \mu_k$, where $|\mu_{HFS}(\varphi)|$ signifies the number of items in $\mu_{HFS}(\varphi)$.

Definition 2.9 (Karamaz and Karaaslan 2021) Let λ'_{RHFPSS} be an RHFPSS of an HFPSS λ_{HFPSS} over the set of parameters Φ_{PS}. The reduced FS (RFS) of λ'_{RHFPSS} is denoted by μ'_{RFS} and defined by

$$\mu'_{RFS} = \{(\psi,\ \mu_{RFS}(\psi)) : \psi \in \Psi_{US}\},$$

where $\mu_{RFS}(\psi) = \frac{1}{|\Phi_{PS}|} \sum_{\varphi \in \Phi_{PS}} r(\mu_{HFS}(\varphi)) \tau_{\lambda_{SS}(\varphi)}(\psi)$ and $\tau_{\lambda_{SS}(\varphi)}(\psi) = \begin{cases} 1, & \psi \in \lambda_{SS}(\varphi) \\ 0, & \psi \notin \lambda_{SS}(\varphi). \end{cases}$

3. KARAMAZ-KARAASLAN DMM BASED ON HFPSS

In this present sec. we have presented the Karamaz-Karaaslan DMM and with the help of one real-life example, we have shown that the Karamaz-Karaaslan DMM is not sufficient to solve HFPSS based DMPs. Karamaz and Karaaslan (2021) proposed the following DMM to solve HFPSS based DMPs.

Algorithm 1 (Karamaz-Karaaslan DMM)

Step1. Construct an HFS $\mu_{HFS} = \{(\varphi,\ \mu_{HFS}(\varphi)) :\ \varphi \in \Phi_{PS}\}$ based on the DMs opinions about the set of parameters Φ_{PS}

Step2. Constructs an HFPSS $\lambda_{HFPSS} = \{((\varphi, \mu_{HFS}(\varphi)), \lambda_{SS}(\varphi)) : \varphi \in \Phi_{PS}\}$ on Ψ_{US} based on opinions of DMs

Step3. Obtain the RHFPSS λ'_{RHFPSS} of λ_{HFPSS}

Step4. Obtain the RFS μ'_{RFS} of λ'_{RHFPSS}

Step5. Select the member from μ'_{RFS}, which has the largest membership value.

In the following real-life example, we have to show that the Karamaz-Karaaslan DMM is not sufficient to solve HFPSS based DMPs.

Example 3.1 Assume that a company, which has four DMsM_1 = Director, M_2 = Executive Director, M_3 = Marketing Officer and M_4 = Finance Manager wish to invest in one of the following companies:

ψ_1 = Microsoft (Information technology firm),

ψ_2 = Ford (Automotive firm),

ψ_3 = Apple (Electronics firm),

ψ_4 = Amazon (Retail & Information technology firm),

ψ_5 = Sinopec Group (Oil & Gas firm),

ψ_6 = Walmart (Retail firm).

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Let us consider the following six parameters:

$\varphi_1 =$ **Innovation**: Consumer expectations and tastes are constantly changing and difficult to forecast. As a result, it is critical for businesses to continue to enhance and reinvent their products. Investors must assess if the company is allocating sufficient money to research and development, the launch of new items in response to client demand, and brand growth through ads.

$\varphi_2 =$ **Management:** It's critical to look at things like who the company's promoters are, how much stock they own, and how active they are in managing the company's affairs. In general, it is a good sign for investors if the promoters hold considerable shares and continue to increase their holding.

$\varphi_3 =$ **Uniqueness:** Investors should look at the company's uniqueness before making a decision. If the company's products or services aren't questioned, it can earn money for a long time. It's possible that the distinctiveness is due to a perceived quality that can't be duplicated by its competitors.

$\varphi_4 =$ **Competition dynamics:** Investors should also consider the dynamics of competition in the areas in which they intend to invest. It is preferable to invest in industries where a new competitor's scope is limited. Companies in industries that demand large initial investments and have greater switching costs for their products or services may be a good long-term investment.

$\varphi_5 =$ **Earnings potential:** Investors must determine whether the products or services of the companies whose stocks they intend to purchase have the potential to raise revenue in the future. Only when there is a long-term demand for the items or services can revenue increase. As a result, it is critical to assess and ensure that the products/services of the firm in which you intend to invest will not become obsolete anytime soon.

$\varphi_6 =$ **Debt-to-Operating Cash Flow:** It informs you how many years a company will be able to repay its debt using cash earned by its operations to do so. When this ratio is high, it indicates that the company will take a long time to pay off its obligations and, as a result, will devote a considerable amount of its profits to this purpose. Shareholders will be left with fewer options.

We consider $\Psi_{US} = \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\}$ be the set of the universe, which contains a collection of firms, and $\Phi_{PS} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6\}$ be the set of parameters associated with the universes mentioned above.

Step1. Assume that DMs construct an HFS μ_{HFS} over the parameters set Φ_{PS} as

$$\mu_{HFS} = \left\{ \begin{array}{l} (\varphi_1, \{0.2, 0.6\}),\ (\varphi_2, \{0.2, 0.3, 0.4, 0.7\}),\ (\varphi_3, \{0.2, 0.4, 0.6\}), \\ (\varphi_4, \{0.2, 0.3, 0.5, 0.6\}),\ (\varphi_5, \{0.3, 0.5\}),\ (\varphi_6, \{0.3, 0.4, 0.5\}) \end{array} \right\}$$

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Step2. DMs construct an HFPSS λ_{HFPSS} on Ψ_{US} as

$$\lambda_{HFPSS}=\begin{Bmatrix}(\varphi_1,^{\{0.2,0.6\}}\{\psi_1,\psi_2\}),\ (\varphi_2,^{\{0.2,0.3,0.4,0.7\}}\{\psi_1,\psi_4,\psi_5,\psi_6\}),\ (\varphi_3,^{\{0.2,0.4,0.6\}}\{\psi_2,\psi_3,\psi_5\}),\\ (\varphi_4,^{\{0.2,0.3,0.5,0.6\}}\{\psi_1,\psi_2,\psi_5,\psi_6\}),\ (\varphi_5,^{\{0.3,0.5\}}\{\psi_3,\psi_4,\psi_6\}),\ (\varphi_6,^{\{0.3,0.4,0.5\}}\{\psi_3,\psi_4\})\end{Bmatrix}.$$

Step3. Now DMs obtain the RHFPSS λ'_{RHFPSS} of λ_{HFPSS} as

$$\lambda'_{RHFPSS}=\begin{Bmatrix}(\varphi_1,^{0.4}\{\psi_1,\psi_2\}),\ (\varphi_2,^{0.4}\{\psi_1,\psi_4,\psi_5,\psi_6\}),\ (\varphi_3,^{0.4}\{\psi_2,\psi_3,\psi_5\}),\\ (\varphi_4,^{0.4}\{\psi_1,\psi_2,\psi_5,\psi_6\}),\ (\varphi_5,^{0.4}\{\psi_3,\psi_4,\psi_6\}),\ (\varphi_6,^{0.4}\{\psi_3,\psi_4\})\end{Bmatrix}.$$

Step4. DMs obtain the RFS μ'_{RFS} of λ'_{RHFPSS} as

$$\mu'_{RFS}=\{(\psi_1,0.2),\ (\psi_2,0.2),\ (\psi_3,0.2),\ (\psi_4,0.2),\ (\psi_5,0.2),\ (\psi_6,0.2)\}$$

Step5. From the above RFS μ'_{RFS}, we can see that all of the membership degrees are the same, 0.2, therefore DMs are unable to choose the best optimal decision in this circumstance. As a result, we may conclude that the Karamaz-Karaaslan method is insufficient for solving DMPs based on HFPSS.

To solve the above DMP, we have modified the Karamaz-Karaaslan Method by introducing two novel operators RMSO and GMO as shown in the next sec.

4. MODIFIED DMM BASED ON HFPSS

In this present sec., we have introduced two novel operators RMSO and GMO on HFEs of HFPSS and deduced the notions of RMS-level-FPSS, GM-level-FPSS, and also deduced their IFSs. Using these new notions, we have proposed an advanced and adjustable DMM based on HFPSS, which is more stable and more feasible than the Karamaz-Karaaslan DMM. Let $\lambda_{HFPSS}=\{((\varphi,\mu_{HFS}(\varphi)),\lambda_{SS}(\varphi)):\varphi\in\Phi_{PS}\}$ be an HFPSS over the set of alternatives Ψ_{US} and Φ_{PS} be a parameter set. Let $d=|\mu_{HFS}(\varphi)|$ denotes the number of values in $\mu_{HFS}(\varphi)$ and $HFE(\Phi_{PS})$ means the collection of all HFEs on Φ_{PS}.

Definition 4.1 The RMSO on $HFE(\Phi_{PS})$ is denoted by Ω_{RMS} and defined by $\Omega_{RMS}:HFE(\Phi_{PS})\to[0,1]$ as $\forall\mu_{HFS}(\varphi)\in HFE(\Phi_{PS})$,

$$\Omega_{RMS}\left(\mu_{HFS}(\varphi)\right)=\sqrt{\frac{1}{d}\sum_{\mu_k\in\mu_{HFS}(\varphi)}(\mu_k)^2}=\left(\frac{1}{d}\sum_{\mu_k\in\mu_{HFS}(\varphi)}(\mu_k)^2\right)^{\frac{1}{2}},$$

where $\Omega_{RMS}\left(\mu_{HFS}(\varphi)\right)$ is called the RMS-value (simply, RMSV) of HFE $\mu_{HFS}(\varphi)$ in the HFPSS λ_{HFPSS}.

Definition 4.2 The GMO on $HFE(\Phi_{PS})$ is denoted by Θ_{GM} and defined by $\Theta_{GM}:HFE(\Phi_{PS})\to[0,1]$ as $\forall\mu_{HFS}(\varphi)\in HFE(\Phi_{PS})$,

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$$\Theta_{GM}\left(\mu_{HFS}(\varphi)\right)=\sqrt[d]{\prod_{\mu_k\in\mu_{HFS}(\varphi)}\mu_k}=\left(\prod_{\mu_k\in\mu_{HFS}(\varphi)}\mu_k\right)^{\frac{1}{d}},$$

where $\Theta_{GM}\left(\mu_{HFS}(\varphi)\right)$ is called the GM-value (simply, GMV) of HFE $\mu_{HFS}(\varphi)$ in the HFPSS λ_{HFPSS}.

Definition 4.3 The RMS-level-FPSS of HFPSS λ_{HFPSS} is denoted by Ω'_{FPSS} and defined as $\Omega'_{FPSS}=\left\{\left((\varphi,\Omega_{RMS}(\mu_{HFS}(\varphi))),\lambda_{SS}(\varphi)\right):\varphi\in\Phi_{PS}\right\}$, where $\Omega_{RMS}\left(\mu_{HFS}(\varphi)\right)$ is RMSV of HFE $\mu_{HFS}(\varphi)$ in the HFPSS λ_{HFPSS}.

Definition 4.4 The GM-level-FPSS of HFPSS λ_{HFPSS} is denoted by Θ'_{FPSS} and defined as $\Theta'_{FPSS}=\left\{\left((\varphi,\Theta_{GM}(\mu_{HFS}(\varphi))),\lambda_{SS}(\varphi)\right):\varphi\in\Phi_{PS}\right\}$, where $\Theta_{GM}\left(\mu_{HFS}(\varphi)\right)$ is GMV of HFE $\mu_{HFS}(\varphi)$ in the HFPSS λ_{HFPSS}.

Definition 4.5 Let Ω'_{FPSS} be the RMS-level-FPSS of an HFPSS λ_{HFPSS} over the parameter set Φ_{PS}. Then, the IFS of Ω'_{FPSS} is denoted by Ω'_{IFS} and defined as $\Omega'_{IFS}=\left\{\left(\psi,\Omega'_{IFS}(\psi)\right):\psi\in\Psi_{US}\right\}$, where

$$\Omega'_{IFS}(\psi)=\frac{1}{|\Phi_{PS}|}\sum_{\varphi\in\Phi_{PS}}\Omega_{RMS}\left(\mu_{HFS}(\varphi)\right)\tau_{\lambda_{SS}(\varphi)}(\psi)\text{ and }\tau_{\lambda_{SS}(\varphi)}(\psi)=\begin{cases}1, & \psi\in\lambda_{SS}(\varphi)\\0, & \psi\notin\lambda_{SS}(\varphi).\end{cases}$$

Definition 4.6 Let Θ'_{FPSS} be the GM-level-FPSS of an HFPSS λ_{HFPSS} over the parameter set Φ_{PS}. Then, the IFS of Θ'_{FPSS} is denoted by Θ'_{IFS} and defined by $\Theta'_{IFS}=\left\{\left(\psi,\Theta'_{IFS}(\psi)\right):\psi\in\Psi_{US}\right\}$, where

$$\Theta'_{IFS}(\psi)=\frac{1}{|\Phi_{PS}|}\sum_{\varphi\in\Phi_{PS}}\Theta_{GM}\left(\mu_{HFS}(\varphi)\right)\tau_{\lambda_{SS}(\varphi)}(\psi)\text{ and }\tau_{\lambda_{SS}(\varphi)}(\psi)=\begin{cases}1, & \psi\in\lambda_{SS}(\varphi)\\0, & \psi\notin\lambda_{SS}(\varphi).\end{cases}$$

4.7 MODIFIED DMM BASE ON HFPSS

Now, we present our advanced and adjustable algorithm for solving DMPs based on HFPSS. The steps of our proposed DMM listed below:

Algorithm 2

Step1. Enter a nonempty universe $\Psi_{US}=\{\psi_1,\psi_2,\psi_3,.....,\psi_m\}$, and a nonempty parameters set $\Phi_{PS}=\{\varphi_1,\varphi_2,\varphi_3,...,\varphi_n\}$

Step2. Construct an HFS μ_{HFS} based on the DMs opinions about the set of parameters Φ_{PS}.

Step3. Construct an HFPSS λ_{HFPSS} on Ψ_{US} according to opinions of DMs.

Step4. Obtain the level-FPSS(RMS-level-FPSS Ω'_{FPSS} or GM-level-FPSS Θ'_{FPSS}) of λ_{HFPSS} (using the operator RMSO Ω_{RMS} or GMO Θ_{GM}).

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Step5. Obtain the IFS of the level-FPSS as in step4.
Step6. Choose the element $\psi_k \in \Psi_{US}$ if the membership value of ψ_k in IFS is maximized.
Step7. If ψ_k has many values, any of ψ_k can be chosen.

Remark 4.8 In the 6th-step of our constructed DMM, one can return to the 4th step and change the operator that the DMs previously used to adjust the best optimal choice, particularly when there are lots of optimal choices to choose from.

5. RESULT AND DISCUSSIONS

In this present Sec., we solve the DMP as in Example 3.1, using our modified DMM, which was not possible to solve using the Karamaz-Karaaslan method.

Example 5.1 Suppose that we consider the DMP as in Example 3.1.

Step1. We consider $\Psi_{US} = \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\}$ be the set of the universe, which contains a collection of firms and $\Phi_{PS} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6\}$ be the set of parameters associated with the universes mentioned above as in Example 3.1.
Step2. Suppose DMs construct an HFS μ_{HFS} on the parameters set Φ_{PS} as

$$\mu_{HFS} = \left\{ \begin{array}{l} (\varphi_1, \{0.2, 0.6\}),\ (\varphi_2, \{0.2, 0.3, 0.4, 0.7\}),\ (\varphi_3, \{0.2, 0.4, 0.6\}), \\ (\varphi_4, \{0.2, 0.3, 0.5, 0.6\}),\ (\varphi_5, \{0.3, 0.5\}),\ (\varphi_6, \{0.3, 0.4, 0.5\}) \end{array} \right\}$$

Step3. DMs construct an HFPSS λ_{HFPSS} on Ψ_{US} as

$$\lambda_{HFPSS} = \left\{ \begin{array}{l} (\varphi_1, {}^{\{0.2,0.6\}}\{\psi_1, \psi_2\}),\ (\varphi_2, {}^{\{0.2,0.3,0.4,0.7\}}\{\psi_1, \psi_4, \psi_5, \psi_6\}),\ (\varphi_3, {}^{\{0.2,0.4,0.6\}}\{\psi_2, \psi_3, \psi_5\}), \\ (\varphi_4, {}^{\{0.2,0.3,0.5,0.6\}}\{\psi_1, \psi_2, \psi_5, \psi_6\}),\ (\varphi_5, {}^{\{0.3,0.5\}}\{\psi_3, \psi_4, \psi_6\}),\ (\varphi_6, {}^{\{0.3,0.4,0.5\}}\{\psi_3, \psi_4\}) \end{array} \right\}.$$

Step4. Now DMs obtain the RMS-level-FPSS Ω'_{FPSS} of λ_{HFPSS} as

$$\Omega'_{FPSS} = \left\{ \begin{array}{l} (\varphi_1, {}^{0.447214}\{\psi_1, \psi_2\}),\ (\varphi_2, {}^{0.441588}\{\psi_1, \psi_4, \psi_5, \psi_6\}),\ (\varphi_3, {}^{0.432049}\{\psi_2, \psi_3, \psi_5\}), \\ (\varphi_4, {}^{0.430116}\{\psi_1, \psi_2, \psi_5, \psi_6\}),\ (\varphi_5, {}^{0.412311}\{\psi_3, \psi_4, \psi_6\}),\ (\varphi_6, {}^{0.408248}\{\psi_3, \psi_4\}) \end{array} \right\}$$

Step5. DMs obtain the IFS Ω'_{IFS} of Ω'_{FPSS} as

$$\Omega'_{IFS} = \left\{ \begin{array}{l} (\psi_1, 0.2198197),\ (\psi_2, 0.2182298),\ (\psi_3, 0.2087680), \\ (\psi_4, 0.2103578),\ (\psi_5, 0.2172922),\ (\psi_6, 0.2140025) \end{array} \right\}$$

Step6. From above IFS Ω'_{IFS}, we see that ψ_1 is the best optimal decision as ψ_1 has the maximized membership value i.e. 0.2198197 and the rank of the firms is
$\psi_1 \succ \psi_2 \succ \psi_5 \succ \psi_6 \succ \psi_4 \succ \psi_3$.

For the flexibility of our constructed DMM, we consider Example3.1 and solve it using GMO as follows:
Example 5.2 We consider the DMP as in Example 3.1.

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Step1. We consider $\Psi_{US} = \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\}$ be the set of the universe, which contains a collection of firms and $\Phi_{PS} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6\}$ be the set of parameters associated with the universes mentioned above as in Example 3.1.

Step2. Suppose DMs construct an HFS μ_{HFS} on the parameters set Φ_{PS} as

$$\mu_{HFS} = \begin{Bmatrix} (\varphi_1, \{0.2, 0.6\}),\ (\varphi_2, \{0.2, 0.3, 0.4, 0.7\}),\ (\varphi_3, \{0.2, 0.4, 0.6\}), \\ (\varphi_4, \{0.2, 0.3, 0.5, 0.6\}),\ (\varphi_5, \{0.3, 0.5\}),\ (\varphi_6, \{0.3, 0.4, 0.5\}) \end{Bmatrix}$$

Step3. DMs construct an HFPSS λ_{HFPSS} on Ψ_{US} as

$$\lambda_{HFPSS} = \begin{Bmatrix} (\varphi_1, ^{\{0.2,0.6\}}\{\psi_1, \psi_2\}),\ (\varphi_2, ^{\{0.2,0.3,0.4,0.7\}}\{\psi_1, \psi_4, \psi_5, \psi_6\}),\ (\varphi_3, ^{\{0.2,0.4,0.6\}}\{\psi_2, \psi_3, \psi_5\}), \\ (\varphi_4, ^{\{0.2,0.3,0.5,0.6\}}\{\psi_1, \psi_2, \psi_5, \psi_6\}),\ (\varphi_5, ^{\{0.3,0.5\}}\{\psi_3, \psi_4, \psi_6\}),\ (\varphi_6, ^{\{0.3,0.4,0.5\}}\{\psi_3, \psi_4\}) \end{Bmatrix}.$$

Step4. Now DMs obtain the GM-level-FPSS Θ'_{FPSS} of λ_{HFPSS} as

$$\Theta'_{FPSS} = \begin{Bmatrix} (\varphi_1, ^{0.34641}\{\psi_1, \psi_2\}),\ (\varphi_2, ^{0.129615}\{\psi_1, \psi_4, \psi_5, \psi_6\}),\ (\varphi_3, ^{0.219089}\{\psi_2, \psi_3, \psi_5\}), \\ (\varphi_4, ^{0.134164}\{\psi_1, \psi_2, \psi_5, \psi_6\}),\ (\varphi_5, ^{0.387298}\{\psi_3, \psi_4, \psi_6\}),\ (\varphi_6, ^{0.244949}\{\psi_3, \psi_4\}) \end{Bmatrix}$$

Step5. DMs obtain the IFS Θ'_{IFS} of Θ'_{FPSS} as

$$\Theta'_{IFS} = \begin{Bmatrix} (\psi_1, 0.101715),\ (\psi_2, 0.1166271),\ (\psi_3, 0.141889), \\ (\psi_4, 0.126977),\ (\psi_5, 0.080478),\ (\psi_6, 0.1085128) \end{Bmatrix}$$

Step6. From above IFS Θ'_{IFS}, we see that ψ_3 is the best optimal decision as ψ_3 has the maximized membership value i.e. 0.141889 and the rank of the firms is

$\psi_3 \succ \psi_4 \succ \psi_2 \succ \psi_6 \succ \psi_1 \succ \psi_5$.

Remark 5.3: We can see from Example 5.1 that when we use RMSO, we get the best possible resultψ_1 In addition, we can show in Example 5.2 that using the GMOwe haveψ_3 is the best optimal decision. As a result,we can observe that when utilizing RMSO and GMO, the final optimal option for DMs is not the same; this is because our selection methods are different.In general, when GMO is used instead of RMSO, the membership grade value of each element in the parameter set is lower.

Advantages 5.4: When we use Algorithm 2, we get fewer object choices, which make it easier for us to make a decision. However, Algorithm 2can be seen as an adjustable approach toHFPSS based decision making because the final optimal decision is related to the operators on membership values in HFE or in other words, the decision criteria used by DMs. For example, if we choose the RMSO in the 4thstep of Algorithm 2, we shall consider the choice value of each object in the RMS-level-FPSS, if another decision criterion such as the GMO is used; we shall consider choice values in the GM-level-FPSS. In general, the choice value of an object in IFSdeduced from RMS-level-FPSS need not coincide with the value in IFS deduced from GM-level-FPSS. Consequently, the optimal objects determined by the RMSO may be different from those selected according to the GMO. As previously stated, many DMPs are inherently humanistic and subjective in character; as a result, there is no single or standard criterion for DMP in an imprecise context. Algorithm 2 is more efficient and acceptable for many real-world applications because of this configurable feature.

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Comparison Analyses

DMMs based on HFSs and SSs have recently received a lot of attention. Recently, Karamaz and Karaaslan (2021) proposed an approach to HFPSS based decision making using score function, with the goal of using HFSs and SSs more effectively for uncertainty issues prevalent in most real-world issues, but it has some limitations in solving HFPSS based DMPs. In this research work, we have shown inExample3.1 that the Karamaz-Karaaslan method (2021) is not sufficient to solve HFPSS based DMPs, but the constructed method in this paper is very advantageous and has no limitations to solve HMFSS based DMPs and we solve this problem with our constructed DMM as shown in Example 5.1.The novelty of our proposed DMM is the concepts of RMSO and GMO rather than score function, which make our DMM more stable and more feasible than the Karamaz-Karaaslan DMM. Moreover, the Karamaz-Karaaslan method (2021) is not adjustable, whereas our DMM is adjustable because if there are lots of optimal choices to be selected in the 6^{th} step, we can return to the 4^{th} step and adjust the operator to get the best outcome. Another difference is that we solve a real-life DMP (Example 5.1) using our suggested DMM, which was impossible to solve using the Karamaz-Karaaslan method (see Example 3.1). As a result, we can say our constructed DMM in this study is more stable and more feasible than the Karamaz-Karaaslan DMM (2021) to solve HFPSS based DMPs.

6. CONCLUSIONS

DMMs based on HFSs and SSs have recently received a lot of attention. Karamaz and Karaaslan (2021) proposed an approach to HFPSS based decision making using score function, with the goal of using HFSs and SSs more effectively for uncertainty issues prevalent in most real-world issues, but it has some limitations in solving HFPSS based DMPs. In this study, we demonstrated with a real-life example that the Karamaz-Karaaslan DMM is insufficient to solve HFPSS based DMPs. As a result, in this paper, we have defined the notions of RMSO and GMO on HFEs in HFPSS and using these two operators we have proposed an advanced and adjustable DMM for solving HFPSS based DMPs. We have also introduced two types of level-FPSSs as RMS-level-FPSS and GM-level-FPSS and deduced their IFS, using our newly defined two novel operators RMSO and GMO. The novelty of our proposed DMM is the concepts of RMSO and GMO rather than score function, which make our DMM more stable and more feasible than the Karamaz-Karaaslan DMM. The second uniqueness is that our DMM can be adjusted, whereas the Karamaz-Karaaslan approach cannot. Another difference is that we solve a real-life DMP using our suggested DMM, which was impossible to solve using the Karamaz-Karaaslan method. Also, some real-life examples are provided to demonstrate the validity of our proposed modified DMM.

In a future study, we will extend this proposed DMM to other real-life applications in the field of pattern recognition and medical diagnostics.

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