



A Convergence Study of Firefly Algorithm

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Abstract: *In Recent era various optimization dilemmas are usually resolved by applying nature stimulated and meta-heuristic method. Firefly algorithm is one of them wherein male fireflies are supervised by two randomly selected female fireflies to give attention on global search, which provides repaid exploration and exploitation. Firefly algorithm is a population based algorithm it is widely used due to its simple search process and easy way to implementation. In this paper, Lightning bug (Firefly) is applied over fourteen bench mark function including multi-model and uni-model for examine the global optimal solution. Convergence analysis has been carried out effectively after various simulation experiments.*

1. INTRODUCTION

Lightning bug (Firefly) is one of the Swarm Intelligence algorithms which are applied in region of improvisation and engineering tasks. Only this algorithm can solve the multidimensional, nonlinear problem and hybridization as well as meta-heuristic. This algorithm is depended on natural behaviour. This is used in economic emissions, load dispatch problem, multilevel image thresh-holding selection, obtaining optimal test, travelling salesman problem, object tracking etc. Many researchers examine the vast verity of real world problems to obtain global optimum solution. Banati and Bajaj [1] analyzed firefly algorithm with various optimization features. Baykasoğlu and Ozsoydan [2] examined firefly algorithm for analyzing the mechanical problems. Chandrasekaran et al [3] developed binary code for firefly algorithm to solve unit commitment problem. Cheung et al [4] introduced adaptive firefly algorithm for analyzing various parameter. Jamil and Yang [5] gave detail literature survey for global optimization problems. Hassanzadeh et al. Kazemzadeh-Parsi et al. [6, 7, 8, and 9] proposed modified firefly algorithm for engineering design optimization problems. Khalil & Wang [10, 11] analyzed global optimum for upgraded firefly algorithm with constrained and unconstrained problems. Yang [14] obtained global optimization for firefly algorithm using Levy flights. Yu [15] developed stepwise strategy for firefly



algorithm. Wang et al. [12] determined numerical optimization applying lightning bugs (Firefly) algorithm for multi group. Jiang et al. [13] suggested enhanced lightning bugs (Firefly) algorithm for light intensity difference to carry out global optima.

Xin-She Yang [2008] has developed Firefly Algorithm which is commonly used in optimization and engineering tasks. This algorithm is stimulated by the brightness behaviour of lightning bugs (Firefly) algorithm. Firefly algorithm considered for obtaining best results for taken their account randomly generated and their brightness is depending on their performance of the objective function. This paper is devised in this manner. Section 2, explained the Firefly Algorithm along with its flow chart and pseudo code. Set of tested benchmark function is provided in section 3. In section 4, numerical illustrations have been focused towards the convergence study of considered benchmark function. Conclusion is given in section 5.

Firefly Algorithm

Fireflies show peculiarity for swarm intelligence through auto covering and decentralized outcome establishing. All fireflies are presumed to be single-sex and gracefulness of the firefly is an act of light intensity, which denotes the indicator of the fitness of a feasible “candidate solution”. Preclusive population of fireflies is created. After this initialization, one of the parameters for fitness is converted and consequently the fitness is obtained for every firefly in the population. Finally, the firefly can be graded and optimal individuals of a result can be accepted for the further step of evaluation/ estimation. The iteration of the steps may be handled by the number of calculations in pre-decided.

Lightning bug (Firefly) algorithm was according to three idealized norms.

1. Lightning bugs are tempted by each other's neglectful/ imprudent gender.
2. The gracefulness of the lightning bugs is related of the flashiness of the lightning bugs and low graceful fireflies will go away to the higher graceful Lightning bug.
3. The flashiness of Lightning bugs is hinge on the objective function.
4. Firefly algorithms are depended on attractiveness formulation and intensity of light. Pleasant appearance of Lightning bugs is comparative to the sharpness of light which is observed by surrounding Fireflies. The distance of two fireflies is considered by p and $B(p)$ is attractive function, which is shown below.

$$B(p) = B_0 e^{-c p^m}; \quad (m \geq 1) \quad (1)$$

Here, B_0 shows the more brightness at $p=0$ and coefficient C indicates absorption amount of light. Practically, C is developed according to the given problem characteristic which is precisely within the range of 0.1-10 from $C \in [0, \infty]$. Moreover, D is characteristic distance between attractive changes in distance. The value of p is initialized in below mentioned form.

$$p = \frac{1}{D^m} \quad (2)$$

The attribute distance for p , is:

$$D = c^{-1} \longrightarrow 1 \quad \text{when } m \longrightarrow A$$

i and j denote the distance between two Fireflies at q_i and q_j is represented as:

$$p_{ij} = \| q_i - q_j \| = \sqrt{\sum_{k=1}^r (q_{i,k} - q_{j,k})^2} \quad (3)$$

Where $q_{i,k}$ and $q_{j,k}$ expressed the i^{th} and j^{th} three dimensional coordinate of k^{th} component of Firefly and r is the dimension level.



Crusade of i^{th} firefly is fascinated to other appealing firefly j^{th} , which establish the relation between the old and updated position of i^{th} firefly, shown below

$$q_i^{t+1} = q_i^t + B_0 e^{-cp_{i,j}^2} (q_j^t - q_i^t) + A\epsilon_i^t \quad (4)$$

Where, $A \in [0, \infty]$ is considered randomization parameter which generate the vector of random numbers in time t .

Pseudo code for Firefly algorithm can be stated as:

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Let us consider,
Objective function  $f(t)$ 
Where  $t=(q_1, q_2, \dots, q_D)^T$ 
With Initialization  $q_i (i = 1, 2, 3, \dots, M)$ 
Calculate Light intensities by  $I_i = f(q_i)$ 
While ( $t < \text{Max Generation } t_{max}$ ) do
for  $i=1$  to  $M$ , all  $M$  fireflies do
for  $j=1$  to  $M$ , all  $M$  fireflies do
if  $I_j > I_i$  then
Move firefly  $i$  toward  $j$ 
end if
end for  $j$ 
end for  $i$ 
Evaluate new solutions  $f(q_i)$ 
Rank the fireflies and find the current global best  $g^*$ 
end while
    
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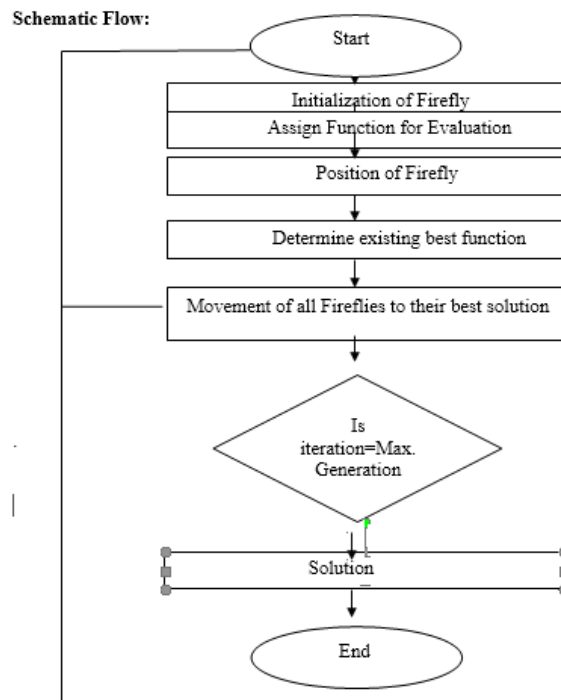


Fig.1 Schematic Flow of Firefly Algorithm



Benchmark Functions

The Lightning bug algorithm is tested over the fourteen benchmark functions. The efficiency of the algorithm depends on its ability how to solve the different set of problems. The nature of the various functions and their diversified range are expressed in table1.

Table 1: The Benchmark test functions

Name	Test Function	Nature of function	Search Space	F_{min}
Ackley	$y = -20e^{-0.02\sqrt{D^{-1}\sum_{i=1}^D q_i^2}} - e^{D-1\sum_{i=1}^D \cos(2\pi q_i)} + 20 + e$	Continuous, differentiable, Non separable, scalable, multi-model	[-32 32]	0
Beale	$y = (1.5 - q_1 + q_1 q_2)^2 + (2.25 - q_1 + q_1 q_2^2)^2 + (2.625 - q_1 + q_1 q_2^3)^2$	Continuous, Differentiable, Non- separable, multi-model	[-4.5 4.5]	0
Booth	$y = (q_1 + 2q_2 - 7)^2 + (2q_1 + q_2 - 5)^2$	Continuous, Differentiable, Non-separable, uni-model	[-10 10]	0
Carron table	$y = - \left[\left\{ \cos(q_1) \cos(q_2) e^{\left 1 - (q_1^2 + q_2^2)^{\frac{0.5}{\pi}} \right } \right\}^2 \right] / 30$	Continuous, Differentiable, Separable, Scalable, Multimodal	[-10 10]	24.1566
Cross in Tray	$y = -0.0001 \left[\left[\frac{\sin(q_1) \sin(q_2)}{e^{\left 100 - (q_1^2 + q_2^2)^{\frac{0.5}{\pi}} \right }} \right]^{0.1} + 1 \right]$	Continuous, Non Separable, Non Scalable, Non-Differentiable, Non-Convex Multimodal	[-10 10]	-2.0626



Easom	$y = -\cos(q_1)\cos(q_2)\exp[-(q_1 - \pi)^2 - (q_2 - \pi)^2]$	Continuous, Non-Convex, Differentiable, separable, multi-model	[-100 100]	-1
Eggholder	$y = -(q_2 + 47) \sin\left(\sqrt{\left q_2 + \frac{q_1}{2} + 47\right }\right) - q_1 \sin(\sqrt{ q_1 - (q_2 + 47) })$	Continuous, Differentiable, Non-separable, multi-model	[-512 512]	-3554.7606
Griewank	$y = \sum_{i=1}^d \frac{q_i^2}{4000} - \prod_{i=1}^d \cos\left(\frac{q_i}{\sqrt{i}}\right) + 1$	Continuous, Non-Convex, Differentiable, separable, Uni-model	[-100 100]	0
Goldstein-price	$y = [1 + (q_1 + q_2 + 1)^2(19 - 14q_1 + 3q_1^2 - 14q_2 + 6q_1q_2 + 3q_2^2)][30 + (2q_1 - 3q_2)^2(18 - 32q_1 + 12q_1^2 + 4q_2 - 36q_1q_2 + 27q_2^2)]$	Continuous, Non-Convex, Differentiable, separable, multi-model	[-2 2]	3
Holdr-table	$y = -\left \sin(q_1)\cos(q_1)\exp\left(1 - \frac{\sqrt{q_1^2 + q_2^2}}{\pi}\right) \right $	Continuous, Non-Differentiable, Non-separable, Non-scalable, Non-Convex, multi-model	[-10 1]	-19.2085
Matyas	$y = 0.26(q_1^2 + q_2^2) - 0.48q_1q_2$	Continuous, Convex, Differentiable, separable, Uni-model	[-10 10]	0
Pen Holder	$y = -\exp\left[\left \left \cos(q_1)\cos(q_2)\right \right _{1 - (q_1^2 + q_2^2)^{\frac{0.5}{\pi}}}\right]^{-1}$	Continuous, Differentiable, Non Separable, Multimodal	[-11 11]	-0.96354



Sphere	$y = \sum_{i=1}^n q_i^2$	Continuous, non-differentiable, separable, Uni-model, Convex, n-dimensional	[-100 100]	0
Schwefel	$y = \sum_{i=1}^n q_i $	Continuous, Differentiable, Non Separable, Unimodal	[-10 10]	0

Numerical illustration

Lightening bug algorithm is more capable to explain the hardest optimization problems. Firefly algorithm is widely used by many researchers due to robustness and best test efficiency. Different variants of Firefly algorithm provide good solution for vast domain of application. The performance of Firefly algorithm random is examined toward their convergence effectively. We consider the common parameters for each Sensitivity analysis reveal that we obtain the best optimal results. Table-2 is depicted the convergence study of benchmark functions.

Table: 2 Convergence values of the Benchmark Functions

Population size=50, Number of iteration= 100,

Functions Name	Algorithms	Mean	Standard Deviation	Dim.	Best	Global Minima
Ackley	GA	14.67178	0.178141	0	0.000	
	PSO	.164622	0.493867			
	Fire- y	.000002	.000001			
Beale	GA	.000000	0.000000	0	0.000	
	PSO	0 .000000	0.000000			
	Fire- y	0.000000	.000000			
Booth	GA	0 .000000	.000000	0	0.000	
	PSO	0.000000	.000000			



	Fire- y	0.000000	0.000000			
CarronTable	GA	5.0876	.7852	0		24.1568
	PSO	5.1546	.9854			
	Fire- y	4.1568	.2135			
Crossin Tray	GA	.2451	.0222	0		2.0626
	PSO	.0154	.1354			
	Fire- y	.0626	.0023			
Easom	GA	1.0000	.000000	0	-1	-1
	PSO	1.0000	.000000			
	Fire- y	1.0000	0.000000			
Eggholder	GA	5561.2143	67.941	0	554.7606	3554.7606
	PSO	217.2235	64.865			
	Fire- y	554.7606	41.778			
Griewank	GA	.00124	.00112	0		
	PSO	.03010	.01211			
	Fire- y	.000015	.000012			
Goldstein- rice	GA	5.2506	2.4617			3
	SO		0	50		
	Fire-fly		0			
Holder-table	GA	.3157	.1034	0		19.2085
	PSO	1.0254	.001			
	Fire- y	9.2085	-5.213			
Penholder	GA	.0684	.0037	0		.96354
	PSO	.2453	.025			
	Fire- y	.96353	.9246			
Matyas	GA	.00021	.00018	0		
	PSO	.00034	.00031			
	Fire- y	.000001	.000001			
Sphere	GA	1.1100	4.2144	0	0.0000	0
	PSO	0.0000	.00000			
	Fire- y	0.0000	.00000			



Schweffel	GA	1.0214	.3868	0	0.0000	
	PSO	0.0000	0.00000			
	Fire- y	0.0000	0.00000			

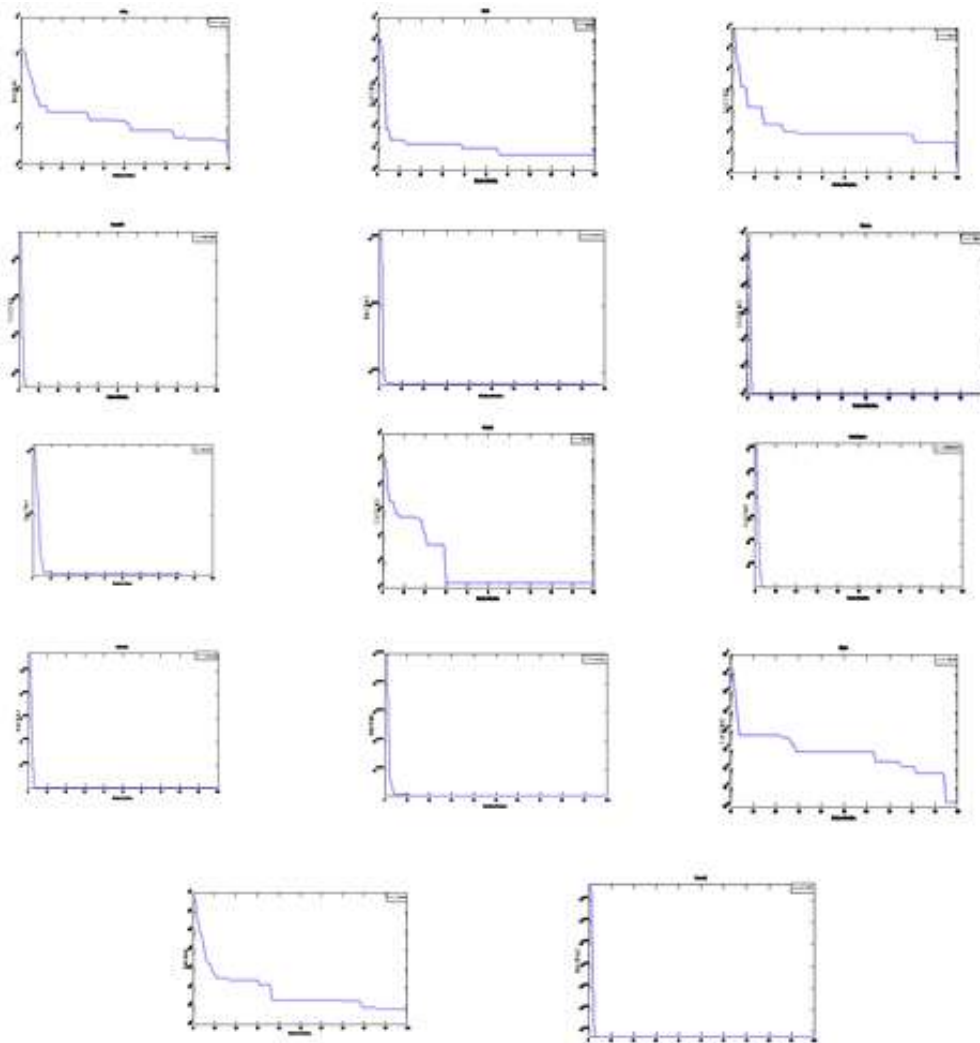


Fig. 2

2. CONCLUSION

In this investigation, we have executed Fire fly Algorithm on fourteen benchmark functions. We have carried out best optimal solution for taking population size 50 and dimension size 50 with respect to the hundred number of iteration. It is observed that the obtained results are more significant to others algorithms. Further, this paper can be extended to boost up the execution of Firefly algorithm by the search of literature.



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